

Ambedkar College of Arts and Science, Wandoor — Class Test 2 (Diagnostic)

Differential Calculus (MAT1CJ101) — B.Sc. Mathematics Honours, Semester I | Module II | Time: 30 Mins | Max Marks: 30

Name: _____ ID No.: _____ Date: _____

8 questions, 5 marks each — answer any 6 (best 6 count, ceiling 30). Show all working; an answer with no justification earns no marks.

Q1. Continuity at a point and on $\mathbb{R}$.

(a) The function f is given by $f(x) = \frac{x^2 - x - 6}{x - 3}$ for $x \neq 3$. How must $f(3)$ be defined for f to be continuous at 3? Verify with the three-part definition. [2 marks]

(b) Find the constants a, b for which g is continuous on $\mathbb{R}$: $g(x) = \begin{cases} 2x + a, & x < 0 \\ x^2 + 3, & 0 \leq x < 2 \\ bx - 1, & x \geq 2 \end{cases}$ [3 marks]

[5 M]

Q2. Continuity on intervals and the Intermediate Value Theorem.

(a) Show that $x^3 - 3x + 1 = 0$ has a root in $(0, 1)$ and another in $(1, 2)$. State exactly what the theorem needs. [2 marks]

(b) Is $f(x) = \sqrt{4 - x^2}$ continuous on $[-2, 2]$? Justify the behaviour at both endpoints. [2 marks]

(c) For $h(x) = \frac{x^2 - 1}{x^2 - 3x + 2}$, classify the discontinuity at $x = 1$ and at $x = 2$. [1 mark]

[5 M]

Q3. The derivative from the definition. Use the limit definition $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ (no differentiation rules).

(a) $f(x) = \sqrt{2x+1}$: find $f'(x)$ and the domain of f' . [3 marks]

(b) $g(x) = \frac{1}{x+2}$: find $g'(x)$. [2 marks]

[5 M]

Q4. Differentiability and continuity.

(a) Let $f(x) = x^2$ for $x \leq 1$ and $f(x) = 3x - 2$ for $x > 1$. Is f continuous at 1? Compute the left and right derivatives and decide if $f'(1)$ exists. [2 marks]

(b) Find the constants a, b for which $h(x) = x^2$ ($x \leq 1$), $h(x) = ax + b$ ($x > 1$) is differentiable at 1. [2 marks]

(c) Show that $y = x^{1/3}$ is continuous at 0 but not differentiable there. [1 mark]

[5 M]

Q5. Differentiation rules and tangent lines.

(a) Differentiate $f(x) = (x^2 + 1)(\sqrt{x} - 3x)$ for $x > 0$. [2 marks]

(b) Find the equation of the tangent to $g(x) = \frac{2x-3}{x^2+1}$ at $x = 1$. [2 marks]

(c) Find the points on $y = x^3 - 6x^2 + 9x + 1$ where the tangent is horizontal. [1 mark]

[5 M]

Q6. Rates of change. A particle moves along a line with position $s(t) = t^3 - 9t^2 + 24t$ metres, $t \geq 0$ seconds.

(a) Find its velocity and acceleration. [1 mark]

(b) When is it momentarily at rest? [1 mark]

(c) Find the total distance travelled during $0 \leq t \leq 5$. [2 marks]

(d) At $t = 1$, is the particle speeding up or slowing down? Justify. [1 mark]

[5 M]

Q7. The chain rule.

(a) Find $\frac{dy}{dx}$ for $y = \left(\frac{2x+1}{x-3}\right)^4$. [3 marks]

(b) Use the table to find $(f \circ g)'(1)$ and $(g \circ f)'(1)$. [2 marks]

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	-3	3	2
3	-1	4	1	-2
5	0	1	2	6

[5 M]

Q8. Implicit differentiation and rational exponents.

(a) Find the tangent to $x^2 + xy + y^2 = 7$ at $(2, 1)$. [2 marks]

(b) If $y = (x^2 + 5)^{3/2}$, find $\frac{dy}{dx}$ at $x = 2$. [1 mark]

(c) For $x^2 + y^2 = 25$, show that $\frac{d^2y}{dx^2} = -\frac{25}{y^3}$. [2 marks]

[5 M]