

Ambedkar College of Arts and Science, Wandoor — Class Test 1

Differential Calculus (MAT1CJ101) — B.Sc. Mathematics Honours, Semester I | Time: 30 Mins | Max Marks: 30

8 questions, 5 marks each — answer any 6 (best 6 count, ceiling 30). Show all working; an answer with no justification earns no marks.

Q1. Let $f(x) = \sqrt{x-1}$ and $g(x) = 4 - x^2$.

- (a) Find $(f \circ g)(x)$, its domain and its range. [2 marks]
 - (b) Find $(g \circ f)(x)$, its domain and its range. [2 marks]
 - (c) A student simplifies $(g \circ f)(x)$ to $5 - x$ and says the function is the same as $h(x) = 5 - x$ on $\mathbb{R}$. Is the student right? Explain. [1 mark]
- [5 M]**

Q2.

- (a) Write $F(x) = (x+1)^3$ as the sum of an even function and an odd function, and identify each. [3 marks]
 - (b) Classify $G(x) = \lfloor x \rfloor + \lfloor -x \rfloor$ as even, odd, both or neither, with proof. [2 marks]
- [5 M]**

Q3. The function f is defined by $f(x) = |x-1|$ for $-1 \leq x \leq 3$. Let $g(x) = 3 - 2f(2x-1)$.

- (a) Find the domain and range of g . [2 marks]
 - (b) Describe, in the correct order, the sequence of shifts, stretches and reflections that carries the graph of f to the graph of g . [1 mark]
 - (c) Find all x with $g(x) = 0$. [2 marks]
- [5 M]**

Q4. Evaluate, justifying each step:

- (a) $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x-4}$ [2 marks]
 - (b) $\lim_{h \rightarrow 0} \frac{\frac{1}{(3+h)^2} - \frac{1}{9}}{h}$ [2 marks]
 - (c) $\lim_{x \rightarrow -1} \frac{x^3 + 1}{x^2 - 1}$ [1 mark]
- [5 M]**

Q5.

- (a) Does $\lim_{x \rightarrow 1} \lfloor x^2 \rfloor$ exist? Compute the one-sided limits. [1 mark]
 - (b) Compute $\lim_{x \rightarrow 0} \lfloor x^2 \rfloor$. [1 mark]
 - (c) Evaluate $\lim_{x \rightarrow 0} x \left\lfloor \frac{1}{x} \right\rfloor$, handling both signs of x . [3 marks]
- [5 M]**

Q6.

- (a) Given that $\lim_{x \rightarrow 1} \frac{f(x) - 5}{x - 1} = 4$, find $\lim_{x \rightarrow 1} f(x)$, justifying with the limit rules. [2 marks]
 - (b) Find the constant k for which $\lim_{x \rightarrow -1} \frac{x^2 + kx + 3}{x + 1}$ exists, and evaluate that limit. [2 marks]
 - (c) If $|f(x)| \leq x^2$ for all x , find $\lim_{x \rightarrow 0} \frac{f(x)}{x}$ and name the theorem you use. [1 mark]
- [5 M]**

Q7.

- (a) Find $\lim_{x \rightarrow \infty} (\sqrt{4x^2 + x} - 2x)$. [2 marks]
 - (b) Find $\lim_{x \rightarrow -\infty} (\sqrt{4x^2 + x} - 2x)$. [1 mark]
 - (c) Evaluate $\lim_{x \rightarrow \infty} \frac{2x + \sin x}{x - \cos x}$, justifying the use of any theorem. [2 marks]
- [5 M]**

Q8. True or False, with a counterexample or a one-line proof. (0.5 mark verdict + 0.5 mark justification each.)

- (a) If $\lim_{x \rightarrow a} f(x)$ exists, then f is defined at a .
- (b) If $\lim_{x \rightarrow a} f(x)$ exists and $\lim_{x \rightarrow a} g(x)$ does not exist, then $\lim_{x \rightarrow a} (f + g)(x)$ does not exist.
- (c) If $\lim_{x \rightarrow a} f(x)g(x) = 0$, then $\lim_{x \rightarrow a} f(x) = 0$ or $\lim_{x \rightarrow a} g(x) = 0$.
- (d) If $f(x) < g(x)$ for all $x \neq a$ and both limits exist, then $\lim_{x \rightarrow a} f(x) < \lim_{x \rightarrow a} g(x)$.
- (e) $\lim_{x \rightarrow 0} \frac{1}{x} = \infty$.

[5 M]