

Ambedkar College of Arts and Science, Wandoor
Class Test 0 — Precalculus Diagnostic — Answer Key
Differential Calculus — MAT1CJ101

B.Sc. Mathematics Honours (CUFYUGP 2024) — Semester I

Maximum Marks: 30

Time: 30 Minutes

Part A — Short Answer ($6 \times 2 = 12$ marks)

1. Reflexivity and symmetry of a relation given by roster. [2]

Answer: R is **reflexive**, since $(1, 1), (2, 2), (3, 3), (4, 4) \in R$ — every element of A is related to itself. R is **not symmetric**, since $(3, 4) \in R$ but $(4, 3) \notin R$.

2. Composition of two functions, both orders. [2]

Answer:

$$(f \circ g)(x) = f(g(x)) = f(x^2) = 2x^2 - 3, \quad (g \circ f)(x) = g(f(x)) = (2x - 3)^2 = 4x^2 - 12x + 9.$$

3. Solving a linear inequality. [2]

Answer: $2x + 5 > 1 \Rightarrow 2x > -4 \Rightarrow x > -2$. Solution set: $(-2, \infty)$.

4. Factoring a quadratic; end behaviour from the leading term. [2]

Answer: (a) $x^2 - 5x + 6 = (x - 2)(x - 3)$. (b) For $f(x) = -2x^3 + x - 1$, the leading term $-2x^3$ dominates for large $|x|$: as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$; as $x \rightarrow -\infty$, $f(x) \rightarrow +\infty$.

5. Laws of exponents; evaluating a logarithm. [2]

Answer: (a) $\frac{a^5 \cdot a^{-2}}{a} = a^{5-2-1} = a^2$. (b) $\log_3 27 = 3$, since $3^3 = 27$.

6. Radian-degree conversion; standard trig value. [2]

Answer: (a) $\frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$. (b) $\tan\left(\frac{\pi}{4}\right) = 1$.

Part B — Applied Problems ($6 \times 3 = 18$ marks)

7. Equivalence classes of a “same remainder mod 3” relation. [3]

Answer: The remainders of $1, \dots, 6$ on division by 3 are: $1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 0, 4 \rightarrow 1, 5 \rightarrow 2, 6 \rightarrow 0$. Grouping elements with equal remainder gives the equivalence classes

$$\{3, 6\}, \quad \{1, 4\}, \quad \{2, 5\}.$$

8. Bijectivity and inverse of a linear function. [3]

Answer: Injective: if $f(a) = f(b)$ then $3a - 2 = 3b - 2 \Rightarrow a = b$. **Surjective:** given any $y \in \mathbb{R}$, take $x = \frac{y+2}{3} \in \mathbb{R}$; then $f(x) = y$. Hence f is bijective. Solving $y = 3x - 2$ for x gives the inverse:

$$f^{-1}(x) = \frac{x+2}{3}.$$

9. Quadratic inequality via sign chart. [3]

Answer: $x^2 - x - 6 = (x - 3)(x + 2) \leq 0$. The critical points are $x = -2, 3$. Since the parabola opens upward, the expression is ≤ 0 between (and including) the roots:

$$x \in [-2, 3].$$

10. Vertical and horizontal asymptotes of a rational function. [3]

Answer: Vertical asymptote: the denominator is zero at $x = -3$, and the numerator $2(-3) - 1 = -7 \neq 0$ there, so $x = -3$ is a vertical asymptote (not a hole). **Horizontal asymptote:** numerator and denominator have equal degree (1), so the horizontal asymptote is the ratio of leading coefficients, $y = \frac{2}{1} = 2$.

11. Logarithmic equation with an extraneous-root check. [3]

Answer: Combine: $\log_2 x + \log_2(x - 2) = \log_2[x(x - 2)] = 3 \Rightarrow x(x - 2) = 2^3 = 8 \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow (x - 4)(x + 2) = 0$, so $x = 4$ or $x = -2$. The domain requires $x > 0$ and $x - 2 > 0$, i.e. $x > 2$. So $x = -2$ is **extraneous** and rejected; only

$$x = 4$$

is valid.

12. Solving a trigonometric equation on $[0, 2\pi)$. [3]

Answer: $2 \sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2}$. On $[0, 2\pi)$, sine equals $\frac{1}{2}$ at the reference angle $\pi/6$ in the first and second quadrants:

$$x = \frac{\pi}{6}, \frac{5\pi}{6}.$$

— End of Solutions —