

Ambedkar College of Arts and Science, Wandoor

Assignment 1

Differential Calculus — MAT1CJ101

B.Sc. Mathematics Honours (CUFYUGP 2024) — Semester I

Section A (3 marks each)

1. Find the domain of

$$f(x) = \frac{1}{\sqrt{x-2}(x+3)}.$$

2. Let

$$f(x) = x^2, \quad g(x) = x - 2.$$

Compute $(f \circ g)(x)$ and $(g \circ f)(x)$. Are they equal?

3. Determine the domain of the composite function $f \circ g$ when

$$f(x) = \sqrt{x}, \quad g(x) = x + 1.$$

Also evaluate

$$f(g(3)) \quad \text{and} \quad f(9).$$

4. Find the equation of the graph of

$$y = x^2$$

shifted 4 units to the right and 3 units upward.

5. Evaluate

$$\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3}.$$

6. Evaluate

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x}.$$

7. For

$$f(x) = \begin{cases} 2x + 1, & x \leq 1, \\ x^2 + 2, & x > 1, \end{cases}$$

find

$$\lim_{x \rightarrow 1^-} f(x) \quad \text{and} \quad \lim_{x \rightarrow 1^+} f(x).$$

Does $\lim_{x \rightarrow 1} f(x)$ exist?

8. Evaluate

$$\lim_{x \rightarrow \infty} \frac{3x^3 - 2}{x^3 + 5x}.$$

9. Show that

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0.$$

Section B (5 marks each)

1. Give an equation for the graph obtained by shifting

$$x = 3y^2$$

upward by 2 units and to the right by 3 units.

Sketch both the original graph and the transformed graph, clearly indicating the direction and magnitude of each transformation.

2. Define the left-hand limit of a function f at x_0 .

Construct an example for which

$$\lim_{x \rightarrow c^-} f(x) \neq \lim_{x \rightarrow c^+} f(x).$$

Explain geometrically what this means.

3. Using the Sandwich (Squeeze) Theorem, find the asymptotes of

$$y = 2 + \frac{\sin x}{x}.$$

Justify your answer.

Section C (8 marks)

1. Find a function f satisfying all of the following conditions:

$$\lim_{x \rightarrow \infty} f(x) = 1,$$

$$\lim_{x \rightarrow -\infty} f(x) = -1,$$

and

$$\lim_{x \rightarrow 1^+} f(x) = 1.$$

- (a) Give an explicit example of such a function.
- (b) Sketch its graph.
- (c) Can such a function be defined at $x = 1$?
- (d) Discuss the possible value(s) of $f(1)$ and explain your answer.

“Mathematics is not just about finding answers; it is about understanding why the answers must be true.”