Jamia Nadwiyya Arts and Science College, Edavanna

Class Test 1 — Answer Key

Mathematical Logic, Set Theory and Combinatorics — MAT1MN104

B.Sc. Mathematics Honours (CUFYUGP 2024) — Semester I (Minor)

Maximum Marks: 30 **Time: 30 Minutes**

Part A — Short Answer (8 questions, 3 marks each; ceiling 18)

1. Membership vs. nesting for $A = \{1, \{2, 3\}, 4\}$. **[3]**

Answer: The elements of A are exactly three objects: 1, $\{2, 3\}$, and 4.

- (a) **False** — 2 is not itself one of A's three elements; it is merely an element of the set $\{2, 3\}$, which is what sits inside A.
- (b) **True** — $\{2, 3\}$ is, as a whole object, literally one of the three elements of A.
- (c) **False** — $|A| = 3$, not 4. A nested set counts as *one* element, however many things are inside it.

2. $\in$ vs. $\subseteq$ vs. power-set membership for $A = \{1, 2, 3\}$. **[3]**

Answer: $\mathcal{P}(A) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.

- (a) **True** — $\{1, 2\}$ is a subset of A, so it is one of the eight elements listed inside $\mathcal{P}(A)$.
- (b) **False** — the elements of A are $1, 2, 3$ individually; $\{1, 2\}$ (a set of two things) is not one of them.
- (c) **False** — the elements of $\mathcal{P}(A)$ are themselves *sets* (subsets of A); the bare number 2 is not a set and does not appear in the list above.

3. Inverse power-set problem: $|\mathcal{P}(A)| = 64$. **[3]**

Answer: $|\mathcal{P}(A)| = 2^{|A|}$, so $2^{|A|} = 64 = 2^6$, giving $|A| = 6$. The number of proper subsets is $2^{|A|} - 1 = 64 - 1 = 63$ (every subset except A itself).

4. Properties of $\subseteq$ vs. $\subset$. **[3]**

Answer:

- (a) **True** — every element of A is (trivially) an element of A, so $A \subseteq A$ always holds. ($\subseteq$ is reflexive.)
- (b) **False** — $A \subset A$ would require $A \subseteq A$ *and* $A \neq A$; the second condition never holds. ($\subset$ is irreflexive.)
- (c) **True** — this is exactly the definition of set equality used throughout the notes: $A = B \iff (A \subseteq B) \wedge (B \subseteq A)$.

5. Well-definedness of a set. **[3]**

Answer: (a) **Not** well-defined — "good" is a subjective judgement; different people could disagree on whether a given teacher belongs to the collection, so membership is not determined. (b) **Well-defined** — "prime and less than 20" is an objective, checkable property; the set is exactly $\{2, 3, 5, 7, 11, 13, 17, 19\}$, unambiguously.

6. Roster form of $\{x \mid x^2 = 9, x \in \mathbb{Z}\}$. **[3]**

Answer: $\{x \mid x^2 = 9, x \in \mathbb{Z}\} = \{-3, 3\}$ — *both* square roots of 9 are integers and must be included. The set is **finite**, with 2 elements.

7. Roster form of $\{n \in \mathbb{Z} \mid n^2 < 20\}$. **[3]**

Answer: Check integers near the boundary: $4^2 = 16 < 20$ (included), $5^2 = 25 \not< 20$ (excluded). So

$$A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}, \qquad |A| = 9.$$

8. $\varnothing$ and A as elements of $\mathcal{P}(A)$. **[3]**

Answer:

- (a) **True** — $\varnothing \subseteq A$ holds for every set A, so $\varnothing$ is always one of the subsets listed in $\mathcal{P}(A)$.
- (b) **False** in general — $\varnothing$ is an *element* of A only if A was specifically built to contain it (e.g. $A = \{\varnothing, 1\}$); for a typical set like $A = \{1, 2\}$, $\varnothing \notin A$.
- (c) **True** — $A \subseteq A$ always (reflexivity, as in Q4), so A itself is always one of the subsets listed in $\mathcal{P}(A)$.

Part B — Medium Answer (4 questions, 6 marks each; ceiling 12)

9. $A = \{1, 2, \{1, 2\}\}$: cardinality, power set, and coexistence of $\in$ and $\subseteq$. **[6]**

Answer:

- (a) A has exactly three elements — 1, 2, and $\{1, 2\}$ (the last is a single nested-set element, not two) — so $|A| = 3$.
- (b) Treating A's three elements as atomic building blocks, $\mathcal{P}(A)$ has $2^3 = 8$ subsets:

$$\varnothing,\ \{1\},\ \{2\},\ \{\{1,2\}\},\ \{1,2\},\ \{1,\{1,2\}\},\ \{2,\{1,2\}\},\ \{1,2,\{1,2\}\}.$$

- (c) **Both are True.** $\{1, 2\} \subseteq A$ because every element of $\{1, 2\}$ — namely 1 and 2 — is individually an element of A. Separately, $\{1, 2\} \in A$ because the whole object $\{1, 2\}$ is literally one of A's three elements. These are two different questions ("are 1 and 2 each in A?" vs. "is the packaged object $\{1, 2\}$ itself in A?") and nothing stops both answers from being yes at once.

10. Construct $A \in B$ and $A \subseteq B$ simultaneously; subset/proper-subset/non-empty counts for $|S| = 6$. **[6]**

Answer:

- (a) Let $A = \{1\}$ and $B = \{1, \{1\}\}$. Then $A \in B$, since $\{1\}$ is literally one of B's two elements. Also $A \subseteq B$, since the one element of A (namely 1) is an element of B. Both hold at once. (Any similarly constructed example is acceptable.)
- (b) For $|S| = 6$: number of subsets $= 2^6 = 64$; number of proper subsets $= 2^6 - 1 = 63$ (excluding S itself); number of non-empty subsets $= 2^6 - 1 = 63$ (excluding $\varnothing$). Both counts come out to 63, but for different reasons — one excludes S, the other excludes $\varnothing$; they are only numerically equal here because each excludes exactly one of the 64 subsets.

11. Finiteness of $A = \{2, 4, 6, 8, 10\}$; $\mathcal{P}(\varnothing)$ vs. $\mathcal{P}(\{\varnothing\})$. **[6]**

Answer:

- (a) A is **finite**: its elements can be counted completely, $|A| = 5$ (a set is finite precisely when it has n elements for some non-negative integer n).
- (b) $\varnothing$ has no elements, so its only subset is $\varnothing$ itself: $\mathcal{P}(\varnothing) = \{\varnothing\}$, which has **1** element. $\{\varnothing\}$ has exactly one element (namely $\varnothing$), so its subsets are $\varnothing$ and $\{\varnothing\}$ itself: $\mathcal{P}(\{\varnothing\}) = \{\varnothing, \{\varnothing\}\}$, which has **2** elements. They are **not equal** — different cardinality (1 vs. 2), and $\mathcal{P}(\{\varnothing\})$ contains an element ($\{\varnothing\}$) that $\mathcal{P}(\varnothing)$ does not.

12. Transitivity of $\subset$ and of $\subseteq$. **[6]**

Answer: True in both cases.

For $\subset$: If $A \subset B$ and $B \subset C$, then in particular $A \subseteq B$ and $B \subseteq C$, so $A \subseteq C$ (transitivity of $\subseteq$, applied twice via an arbitrary-element argument). It remains to rule out $A = C$: if $A = C$, then combined with $A \subseteq B \subseteq C = A$ we would get $B \subseteq A$ as well, hence $A = B$ (using $A \subseteq B$ and $B \subseteq A$) — but $A \subset B$ requires $A \neq B$, a contradiction. So $A \neq C$, and therefore $A \subset C$.

For $\subseteq$: If $A \subseteq B$ and $B \subseteq C$, then every element of A is in B, and every element of B is in C; hence every element of A is in C, i.e. $A \subseteq C$. This is immediate transitivity, with no extra case to check (no non-equality condition to preserve).

— End of Solutions —