Jamia Nadwiyya Arts and Science College, Edavanna
Class Test 1

Mathematical Logic, Set Theory and Combinatorics — MAT1MN104

B.Sc. Computer Science Honours (CUFYUGP 2024) — Semester I (Minor)

Name: ______________________ ID No.: ______________________

Date: ______________________ Time: 30 Mins **Maximum Marks: 30**

Each section has more questions than its ceiling — you do not need to attempt every question. Show all working for Part B.

Part A — Short Answer *Answer as many as you can. Each carries 3 marks. Overall ceiling: 18 marks.*

1. Let $A = \{1, \{2, 3\}, 4\}$. State True or False, with a one-line reason for each: (a) $2 \in A$ (b) $\{2, 3\} \in A$ (c) $|A| = 4$. **[3]**

2. Let $A = \{1, 2, 3\}$. State True or False, with a one-line reason for each: (a) $\{1, 2\} \in \mathcal{P}(A)$ (b) $\{1, 2\} \in A$ (c) $2 \in \mathcal{P}(A)$. **[3]**

3. If $|\mathcal{P}(A)| = 64$, find $|A|$. Hence state how many of these subsets are *proper* subsets of A. **[3]**

4. State True or False, with a one-line reason for each: (a) $A \subseteq A$ for every set A (b) $A \subset A$ for every set A (c) If $A \subseteq B$ and $B \subseteq A$, then $A = B$. **[3]**

5. Determine, with a one-line reason, whether each of the following is a well-defined set: (a) "the collection of good mathematics teachers" (b) "the collection of prime numbers less than 20". **[3]**

6. Write $\{x \mid x^2 = 9,\ x \in \mathbb{Z}\}$ using the roster method. Is this set finite or infinite? **[3]**

7. Let $A = \{n \in \mathbb{Z} \mid n^2 < 20\}$. List the elements of A using the roster method, and state $|A|$. **[3]**

8. State True or False, with a one-line reason for each: (a) $\varnothing \in \mathcal{P}(A)$ for every set A (b) $\varnothing \in A$ for every set A (c) $A \in \mathcal{P}(A)$ for every set A. **[3]**

Part B — Medium Answer

Answer as many as you can. Each carries 6 marks. Overall ceiling: 12 marks.

9. Let $A = \{1, 2, \{1, 2\}\}$.

(a) Find $|A|$.

(b) List all elements of $\mathcal{P}(A)$.

(c) State True or False: $\{1, 2\} \subseteq A$, and $\{1, 2\} \in A$. Explain why both can hold at the same time.

[6]

10.

(a) Give an example of sets A and B such that $A \in B$ *and* $A \subseteq B$ both hold. Verify your example satisfies both.

(b) A set S has 6 elements. Find the number of subsets of S, the number of proper subsets of S, and the number of non-empty subsets of S.

[6]

11. Let $U = \{1, 2, \ldots, 10\}$ and $A = \{2, 4, 6, 8, 10\}$.

(a) Is A finite or infinite? Justify using the definition of cardinality.

(b) List $\mathcal{P}(\varnothing)$ and $\mathcal{P}(\{\varnothing\})$. State how many elements each has, and whether they are equal as sets.

[6]

12. Determine, with justification, whether the following statement is True or False:

"If $A \subset B$ and $B \subset C$, then $A \subset C$."

Then determine whether the statement remains true if all three uses of $\subset$ are replaced by $\subseteq$. **[6]**

— End of Question Paper —