

Name
Reg. No.

AMBEDKAR COLLEGE OF ARTS AND SCIENCE, WANDOOR
FIFTH SEMESTER (CUFYUGP) DEGREE EXAMINATION
SEPTEMBER 2026

Department of Mathematics

MAT5EJ302(1)—Data Structures and Algorithms
(2024 Admission onwards)

Time: One and half hours

Maximum: 50 Marks

Section A

All questions can be answered.
Each question carries 2 marks.

(Ceiling: 16 Marks)

1. What is $2^{2006} \bmod 3$? Justify your answer in one line.
2. State Euclid's rule $\gcd(a, b) = \gcd(b, a \bmod b)$, and use it to compute $\gcd(84, 30)$, showing every step.
3. Compute $(43 \times 61) \bmod 12$ by first reducing 43 and 61 modulo 12.
4. Show that the binary representation of an integer is at most about four times as long as its decimal representation. For very large numbers, what is the exact ratio of these two lengths?
5. The straightforward way to test whether N is prime checks every candidate divisor up to $\sqrt{N}$. Explain in one line why this is exponential in the number of bits n of N .
6. Write the recurrence relation for binary search on a sorted array of n elements, and state its solution.
7. Using the Master Theorem, solve $T(n) = 8T(n/2) + \Theta(n^3)$, naming the case used.
8. A graph has n vertices and m edges. State the storage space required by (a) an adjacency matrix and (b) an adjacency list.
9. In an undirected graph the vertex degrees satisfy $\sum_v \deg(v) = 2|E|$. Using this identity, explain why the number of vertices of odd degree must be even.
10. Define a directed acyclic graph (DAG). State an equivalent characterisation of a DAG in terms of the back edges of a depth-first search.

Section B

All questions can be answered.
Each question carries 6 marks.

(Ceiling: 24 Marks)

1. For each of $5 \bmod 23$, $21 \bmod 91$, and $20 \bmod 79$: state and check the criterion for a multiplicative inverse to exist; then, whenever it exists, compute it using the extended Euclidean algorithm and verify your answer.

2. Is $5^{30,000} - 6^{123,456}$ a multiple of 31? Justify your answer using Fermat's Little Theorem.
3. (a) State Fermat's Little Theorem. (b) Apply the Fermat primality test with base $a = 2$ to $N = 15$ and to $N = 17$, and say what each result establishes. (c) In one line each, explain what a *Fermat liar* and a *Carmichael number* are.
4. Consider the undirected graph on vertices $\{A, B, C, D, E, F\}$ with edges
 $A-B, \quad B-C, \quad A-D, \quad C-E, \quad D-E, \quad E-F$.

Run depth-first search starting from A , visiting neighbours in alphabetical order. Give the pre- and post-numbers of every vertex, classify every edge as a tree edge or a back edge, and state the number of connected components.

5. You must choose among three algorithms for the same problem:
 - **Algorithm A** solves 5 subproblems of half the size, with a linear-time combine step;
 - **Algorithm B** solves 2 subproblems of size $n - 1$, with a constant-time combine step;
 - **Algorithm C** solves 9 subproblems of size $n/3$, with a $\Theta(n^2)$ combine step.

Find the running time of each (with justification), and say which algorithm you would choose.

6. Use the divide-and-conquer (Karatsuba) multiplication algorithm to multiply the binary integers $x = 1110_2$ and $y = 1011_2$. Show the halves x_L, x_R, y_L, y_R , the three recursive products P_1, P_2, P_3 , the combination step, and the final product in decimal and in binary.

Section C

*Answer any one question.
 The question carries 10 marks. $1 \times 10 = 10$ marks*

1. (a) Express 47 in binary.
 (b) Compute $9^{47} \bmod 25$ by repeated squaring: find $9^1, 9^2, 9^4, 9^8, 9^{16}, 9^{32} \pmod{25}$ by successive squaring, then combine the powers needed (per the binary expansion of 47) to get the final answer.
 (c) If y has n bits, how many modular squarings does this method need in the worst case, and how many modular multiplications in total (squarings plus combination steps)? Give your answer in terms of n , and contrast it with the $y - 1$ multiplications the naive method would need.
2. Run the DFS-based topological-sort algorithm, visiting vertices and neighbours in alphabetical order, on the directed graph with vertices $\{A, B, C, D, E, F, G\}$ and edges

$$A \rightarrow B, \quad A \rightarrow C, \quad B \rightarrow D, \quad C \rightarrow D, \quad C \rightarrow E, \quad D \rightarrow F, \quad E \rightarrow F, \quad F \rightarrow G.$$

- (a) Give the pre- and post-numbers of every vertex.
- (b) Classify every edge as tree, forward, back, or cross.
- (c) List all sources and all sinks.
- (d) Give the topological order obtained (by decreasing post-number).