CALCULUS

Lecture 16 — Course Recap

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Recap: Differential Equations (Lectures 10–11)

Lecture 10 — First-Order ODEs

- Classification: order, linearity, homogeneity
- Separable equations: $\frac{dy}{dx} = g(x)h(y) \Rightarrow \int \frac{dy}{h(y)} = \int g(x)\,dx$
- Existence & uniqueness; initial value problems (IVPs)

Lecture 11 — Higher-Order Linear ODEs

- Integrating factor: $\mu(x) = e^{\int P\,dx}$ for $y' + Py = Q$
- Constant-coefficient 2nd-order: auxiliary equation $am^2 + bm + c = 0$
- Three cases: two real roots, repeated root, complex conjugate roots

RECAP: LAPLACE TRANSFORMS (LECTURES 12–13)

Lecture 12 — Laplace Transforms

- Definition: $\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$
- Standard table: $\mathcal{L}\{e^{at}\} = \frac{1}{s-a},\ \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}},\ \mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2}$
- Derivative rule: $\mathcal{L}\{y''\} = s^2 Y - sy(0) - y'(0)$
- Solve IVPs: transform $\to$ solve for $Y(s)$ $\to$ partial fractions $\to$ invert

Lecture 13 — Advanced Laplace

- First Translation: $\mathcal{L}\{e^{at} f(t)\} = F(s-a)$
- Unit Step: $u(t-a)$; Second Translation: $\mathcal{L}\{f(t-a)\,u(t-a)\} = e^{-as}F(s)$
- Convolution Theorem: $\mathcal{L}\{f * g\} = F(s)\cdot G(s)$

RECAP: NUMERICAL METHODS & PDES (LECTURES 14–15)

Lecture 14 — Numerical Methods for ODEs

- Euler: $y_{n+1} = y_n + hf(x_n, y_n)$; global error $O(h)$
- Improved Euler (Predictor-Corrector): global error $O(h^2)$
- Runge-Kutta RK4: four slope samples; global error $O(h^4)$
- Adams-Bashforth-Moulton: multi-step; $O(h^4)$, 2 evals/step after startup

Lecture 15 — Partial Differential Equations

- Classification: $B^2 - 4AC \gtrless 0$ (Hyperbolic / Parabolic / Elliptic)
- Lagrange's method for $Pp + Qq = R$: auxiliary equations $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$
- Separation of variables: $u = X(x)Y(y)$; three λ-cases
- Fundamental equations: Heat ($u_t = ku_{xx}$), Wave ($u_{tt} = \alpha^2 u_{xx}$), Laplace ($u_{xx} + u_{yy} = 0$)

Thank you :)